Modeling the day-to-day traffic evolution process after an unexpected network disruption

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ABSTRACT

Although various approaches have been proposed for modeling day-to-day traffic flow evolution, none of them, to the best of our knowledge, have been validated for disrupted networks due to the lack of empirical observations. By carefully studying the driving behavioral changes after the collapse of I-35W Mississippi River Bridge in Minneapolis, Minnesota, we found that most of the existing day-to-day traffic assignment models would not be suitable for modeling the traffic evolution under network disruption, because they assume that drivers’ travel cost perception depends solely on their experiences from previous days. When a significant network change occurs unexpectedly, travelers’ past experience on a traffic network may not be entirely useful because the unexpected network change could disturb the traffic pattern greatly. To remedy this, in this paper, we propose a prediction-correction model to describe the traffic equilibration process, in which travelers predict traffic patterns after network changes and gradually correct their predictions according to their travel experience. We also prove rigorously that, under mild assumptions, the proposed prediction-correction process has the user equilibrium flow pattern as a globally attractive point. The proposed model is calibrated and validated with the field data collected after the collapse of I-35W Bridge. This study bridges the gap between theoretical modeling and practical applications of day-to-day traffic equilibration approaches and furthers the understanding of traffic equilibration process after network disruption.

Key Words: Day-to-day, traffic equilibration, user equilibrium, network disruption.

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1. INTRODUCTION

Prediction of day-to-day traffic flow evolution is vital for the prioritization of traffic restoration projects after a network disruption, such as the unexpected collapse of the I-35W Mississippi River Bridge in Minneapolis, Minnesota. From the traffic management’s perspective, for a disrupted network, it is essential to understand how traffic pattern evolves from a disequilibrium state toward a new equilibrium, before traffic restoration projects can be planned accordingly.

Day-to-day (or inter-periodic) traffic assignment methods are believed to be most appropriate for analyzing traffic equilibration processes because of their flexibility to accommodate a wide range of behavior rules, levels of aggregation, and traffic models (Watling and Hazelton, 2003). Watling (1999) classified day-to-day traffic assignment models into four types, by whether they are continuous or discrete, and deterministic or stochastic. Existing continuous time day-to-day dynamics employ differential equations to describe traffic evolution. In this category, Smith (1984), Friesz et al. (1994), and Zhang and Nagurney (1996) proposed three dynamical systems. These three systems adopted the assumption of perfect perception of travel cost and developed deterministic traffic assignment processes over a continuous temporal dimension. In discrete time day-to-day traffic dynamical systems, travelers’ route choice behavior is assumed to be repeated daily, in accordance with daily changes in traffic flows. Specifically, Friesz et al. (1994) employed a projection-type discretization algorithm, given by Bertsekas and Gafni (1982), to approximate the continuous traffic trajectories in the dynamical system developed therein. Nagurney and Zhang (1997) specified their continuous model in a discrete temporal space with fixed demand and applied Euler’s method to solve the projected dynamical system. To accommodate the drivers’ route choice stochasticity, existing stochastic day-to-day traffic assignment models followed Markov processes, as those in Cascetta (1989) and Hazelton and Watling (2004). To solve these models, Davis and Nihan (1993) provided a particular Gaussian multi-variant autoregressive process and Hazelton et al. (1996) proposed a Markov Chain Monte Carlo method. The stability property of the day-to-day models has been studied in Horowitz (1984), Cantarella (1993), Cantarella and Cascetta (1995), and Watling (1999).

As noted by Mahmassani (1990), it is difficult to obtain observational evidence of real-world traveler choice behaviors to verify a day-to-day traffic assignment model. Most studies on day-to-day dynamic traffic modeling (e.g., Mahmassani et al., 1986; Jotisankasa and Polak, 2005) rely on experimental approaches rather than field data. Other studies (e.g., Chang and Nojima, 2001; Hunt et al., 2002) provide empirical observations of traffic fluctuation under network disruption.
but no mathematical models are established. Due to the lack of data, very few studies have compared the outcomes of a day-to-day traffic assignment model against reality, and thus the quality of existing models has not been verified. As noted by Friesz and Shah (2001), the urgent need for day-to-day traffic dynamic modeling is neither to establish delicate mathematical formula nor to develop computational tools, but rather “to gather data which allows construction and calibration of the kind of day-to-day adjustment dynamics”. There clearly exists a gap between day-to-day traffic flow evolution models and their practical applications, especially for a disrupted network that are appealing to traffic management authorities.

This paper is devoted to bridging the gap by developing and validating a new discrete-time deterministic day-to-day traffic assignment model, based on the observations collected after the collapse of I-35W Mississippi River Bridge on August 1, 2007. Since the I-35W highway bridge is a major artery in the Twin Cities highway network, its collapse constitutes a significant disruption to the trip-making patterns in the region. While catastrophic to those affected by the collapse in terms of fatalities, injuries, and loss of personal property, it provides a rare research opportunity to observe the traffic flow evolution process empirically. Therefore, the main goal of this work is to gain insights into the traffic flow evolution process given the real-world example.

To verify the proposed day-to-day traffic flow evolution model, two primary sources of information responding to the I-35W bridge collapse are synthesized: the results from travel behavior surveys and loop detector data from the Twin Cities freeway system.

Based on the abovementioned data, we have found that most of the existing day-to-day traffic assignment models would not be suitable for modeling the traffic evolution under network disruption, because they assume that drivers’ cost perception depends solely on their experiences from previous days. For instance, the perception updating process in Nagurney and Zhang (1997) and Yang and Liu (2007) assumes that travelers’ route choice depends on their experienced travel costs on the previous day only; other studies, e.g., Chang and Mahmassani (1988), Cascetta (1989), Davis and Nihan (1993), adopt a weighted average based cost updating mechanism. Such cost updating mechanisms can be considered as cost correction processes, but cannot be applied directly to a traffic network with a significant unexpected disruption. When a significant network change occurs unexpectedly, travelers’ past experience on a traffic network may not be entirely useful because the unexpected network change could disturb the traffic pattern greatly.

The proposed day-to-day traffic assignment model differs from the traditional ones in that the cost updating process considers driver’s forward-looking behavior responding to network disruption. We assume that drivers make predictions of future traffic conditions due to network...
topology changes. The forward-looking responses can be modeled by introducing a predictive component into cost perception, such that the perceived cost pattern involves driver’s anticipated congestion resulting from network disruption. The predictive flow patterns are then corrected with driver’s actual experiences. We name this cost updating procedure as a “prediction-correction” process. By comparing the model results against field data observed before and after the I-35W Bridge collapse, we demonstrate that the prediction component in the model plays a crucial role in replicating the traffic recovery characteristics due to an unexpected network disruption.

The remainder of this paper is organized as follows. The next section exhibits the empirical observations we collected after the collapse of the I-35W Bridge. The empirical data provides us supportive evidence for developing the “prediction-correction” framework, which is discussed in Section 3. Following the framework, Section 4 provides detailed mathematical formulation of a deterministic day-to-day assignment model. Section 5 demonstrates the capability of the proposed model by validating the calibrated day-to-day traffic assignment model against the field data. The final section offers some concluding remarks and future research directions.

2. EMPIRICAL OBSERVATIONS

Our empirical observations are from two data sources: drivers’ behavioral surveys and loop detector data from the Twin Cities freeway system.

**Survey Data**

Two surveys were conducted to better understand travelers' behavior adjustment responding to the I-35W Bridge collapse. The first survey was conducted in the middle of September 2007, six weeks after the collapse. The survey questions were designed to reveal subjects’ day-to-day travel choices for morning commutes and the factors underlying their choices, before and after the bridge collapse. The subjects of the survey were travelers who may have used the I-35W Bridge on a daily basis or who may have used routes that were indirectly affected by the collapse. We distributed about 1000 questionnaires in Downtown Minneapolis and at the University of Minnesota Minneapolis campus, both of which were affected greatly by the bridge collapse. The total number of responses in the survey is 148.

Results from the survey show that the bridge collapse affected travelers' route choices significantly. By comparing the respondents’ morning commute routes before and after the bridge collapse, we found that 31 respondents altered their daily morning commute routes on the day
immediately after the bridge collapse. Interestingly, 14 out of these 31 respondents (about 45%), who changed their daily commuting routes, did not use the I-35W Bridge regularly before it collapsed.

The other internet-based travel survey revealed similar findings. This survey, conducted in November 2008, was designed to study the travel behavior differences for the I-35W Bridge collapse and its reopening in September 2008. We sent 5000 invitations to random individuals within the Twin Cities seven-county metropolitan area and asked them to answer questions associated with their driving behavior changes due to the bridge collapse and reopening. From the survey results, we found that 52 out of 349 respondents changed their daily morning commute routes immediately after the bridge collapse. More than half of those (28 out of 52) who changed their daily morning commute routes were not regular I-35W Bridge users. In addition, 21 of them clarified that “anticipation of worse traffic conditions” was one of the main reasons for route changing.

**Loop Detector Data**

Travelers’ route choice change, at aggregate level, can be also revealed from the traffic volume evolution on the Twin Cities freeway network before and after the collapse of I-35W Bridge. Here we use three cordons, as illustrated by Figure 1, to define three studying areas around the I-35W Bridge. The first cordon, shown as a blue solid circle, has a radius about half-mile. It covers the immediate adjacent area of I-35W Bridge. Vehicles approaching to the bridge have no alternative freeway routes inside this cordon, and thus have to exit. The second cordon, shown as a red dash-dot circle, has a radius of 2.5 miles. This cordon includes I-94 and Trunk Highway 280 (TH 280), which was designated as the detour route after the bridge collapse. The third cordon, shown as a 5.5-mile-radius green dot circle, covers the Minneapolis Central Business District and the City of Minneapolis. Major freeway routes accessing into the Minneapolis Central Business District are included. Loop detectors around the cordon lines are shown as red dots in Figure 1.
We aggregate the inbound traffic volumes from the loop detectors around the three cordons for the morning peak period (between 6 a.m. and 9 a.m. local time). Only non-holiday weekdays are considered. Figure 2 illustrates the daily traffic evolution between Monday, July 9th, 2007 and Friday, November 16th, 2007. As we can observe from the figure, the traffic counts on these three cordons are fairly stable prior to the collapse of I-35W Bridge. However, immediately after the collapse, the traffic counts at cordons drop significantly. The total number of vehicles entering into Cordon 1 decreases to nearly zero, while the numbers of vehicles entering into Cordon 2 and Cordon 3 decrease 40 percent and 20 percent, respectively. In weeks after the collapse, the traffic counts at these three cordons present a recovery process. Without alternative routes, the demand at Cordon 1 stabilizes to a point that is about 50 percent lower than the demand before the collapse. It takes about eight weeks to recover the demand at Cordon 2 to the pre-collapse level. The full recovery time of demand at Cordon 3 is about four weeks which is much shorter than that at Cordon 2. Figure 2 demonstrates that the dramatic demand decreases, or “shocks”, diminish in intensity as alternatives become available, generally corresponding to the distance from the bridge increasing.
We believe that the traffic volume recovery on cordon is mainly due to drivers’ route choice adjustment after the collapse, instead of origin-destination demand changes. By summing up the traffic counts from the loop detectors located on all the on-ramps in the Twin Cities area, we can analyze the daily demand change before and after the bridge collapse. Figure 3 shows the daily demand entering the Twin Cities freeway system, during morning peak period, between July 23rd, 2007, and August 31st, 2007. As shown by the figure, freeway traffic demand in the days and weeks following the bridge collapse does not experience any drastic changes, fluctuating within the bounds of weekly variation. The lone exception is on Thursday, August 2nd, where a noticeable decrease occurs when compared to other Thursdays, but otherwise demand remains consistent with previous weeks. This shows that the bridge collapse did not influence freeway demand during the AM Peak on the system as a whole.

In summary, the empirical evidence from both the traveler surveys and the loop detector data reveal that, after an unexpected network disruption, travelers undergo a learning and exploration process of the new network. Following the I-35W Bridge collapse, drivers were observed to drastically avoid areas near the disruption site until the perceived congestion in that area gradually diminishes. Even for those drivers, who were not regular bridge users, they changed their daily routes because they anticipate worse traffic conditions on these routes due to the diverted traffic from the bridge. This attests that drivers' prediction on a disrupted network plays a
crucial role in their route choice decision-making and this cannot be ignored when we model the
day-to-day traffic flow evolution process after network disruption.

Figure 3 Daily Traffic Entering the Twin Cities Freeway Network via On-Ramps (6-9a.m.)


Suppose that we consider the traffic flow evolution process on a given directed network, denoted
by $G(N, L)$ with node set $N$ and link set $L$. A subset of $N$ constitutes the network centroids,
which generate and attract trips. Throughout this paper, we assume that day-to-day traffic
demands are constant. The total number of trips (or demand) for OD pair $w$ is represented by $d_w$.
The demand vector for all OD pairs is denoted by $d$. A set of paths, denoted by $R^w$, connect one
origin-destination (OD) pair indexed by $w$. The flow on route $r \in R^w$ on day $t$ is denoted by $f_{rt}^t$. All path flows on day $t$ are represented by the vector $f^t$. Let $x_a^t$ represent flow on link $a \in L$ on
day $t$. The vector of link flows is denoted by $x^t$. Let $A = \left( \delta_{ap} \right)$ represent the link-path incidence
matrix, then $x^t = Af^t$. Let $\Phi = \left( \phi_{wp} \right)$ represent the OD-path incidence matrix, then $d = \Phi f^t$.
Denote $C^t$ as the link cost vector on day $t$, with individual link cost $C_a^t$. If we assume that the
link costs are functions of the link flow vector, then $C^t = C \left( x^t \right)$, where $C(\bullet)$ is a given
deterministic function. Let us denote $\hat{x}^t$ as the predicted link flow pattern on day $t$, and $P^t$ the
predicted link cost, then $P^t = C \left( \hat{x}^t \right)$. Finally, denote the vector of driver pre-trip perceived link
cost on day $t$ as $F^t$. 

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Traditional day-to-day traffic assignment models update drivers’ daily cost perception by a “correction-only” process, which combines drivers’ latest experience with their perception on previous days. When a network disruption happens on day $t$, traditional cost updating processes would not reflect the cost perception change resulting from drivers’ forward-looking responses to the disruption. However, the empirical evidence presented in the last section demonstrates that drivers would adjust their route choices based on their predicted traffic flow pattern for a disrupted network, even though they do not have any experience of the new network yet.

Actually, psychologists have already noticed human behavioral choice adjustment can depend on their prediction of system status. As noted by Heath (2000), “human behavior is clearly not random simply because people can predict to some extent how other people will behave”. Following this statement, we formulate drivers’ anticipation of the traffic condition as a prediction process responding to a disruption, and integrate the prediction process with drivers’ experience. Note that, with the prediction component, traffic flows could be altered immediately after a disruption, even though drivers have not traveled in the new network yet. This is a significant contribution of this paper, since, in the existing day-to-day traffic assignment models, travelers cannot change their route choices until they experience the congestion. The proposed “prediction-correction” framework is illustrated in Figure 4.

Figure 4 The Prediction-Correction Framework for Day-to-Day Traffic Assignment
This framework differs from traditional day-to-day traffic assignment models in two aspects. First, the pre-trip travel cost perception updating process includes two major components: Prediction and Correction. The prediction process provides a predicted link flow pattern \( \hat{x}' \) resulting from drivers’ anticipated congestion due to the network disruption. Secondly, the proposed framework differs from traditional models by adopting a link-based network loading, recently proposed by He et al. (2010). Path-based network loading approaches employed by traditional day-to-day traffic models suffer two unavoidable shortcomings: the path-flow non-uniqueness and the path overlapping problem. These issues have been discussed in He et al. (2010).

The calculation of predicted link flows is illustrated by Figure 5. A network disruption will initiate the calculation of predicted traffic flow pattern. The predicted flow pattern is computed by redistributing all impacted traffic to paths with shortest distance. After the first day of network disruption, the predicted flow pattern will be updated by combining actual link flows assigned. We name it as a prediction damping process, since the impact of initial predicted flow pattern will be decreasing over time.

**Figure 5 The Proposed Prediction Process**

4. **MATHEMATICAL FORMULATION**

As in most of the studies on traffic equilibrium, we assume that drivers are rational in terms of route choice and they have perfect information about link costs from previous days. Since the
proposed model involves driver’s prediction of future traffic, we further assume that drivers' prediction is gradually vanishing if no additional network disruption occurs. In other words, the "prediction-correction" framework proposed in this paper will collapse into the traditional "correction-only" framework eventually.

4.1 The Prediction Process

As shown in Figure 5, when a network disruption occurs, we introduce a prediction step into the cost updating process to capture drivers’ anticipation of traffic congestion. Assume a network disruption happens on day 0. Then the initial predicted link flow pattern of the following day $\hat{x}^1$ is a result of reassigning traffic flows impacted by the disruption. In the subsequent days after day 1, the predicted link flow pattern $\hat{x}^t$ is a weighted average between the previous day’s predicted flow pattern $\hat{x}^{t-1}$ and drivers’ actually experienced flow pattern $x^{t-1}$ (note that $t > 1$).

Assume that only those drivers directly impacted by the disruption would switch their current routes to the fastest path in the disrupted network. The initial predicted flows $\hat{x}^1$ can be estimated by reassigning the impacted flows, which follows an all-or-nothing assignment. Mathematically, the predicted flow pattern $\hat{x}^1$ on day 1 is an optimal solution to the minimization problem:

$$\min_{\hat{x} \in \Omega^1} C^0 \hat{x}$$

where $C^0$ represents the free flow link travel cost vector, $\hat{x}$ represents the impacted link flow due to the network topology changes, and $\Omega^1$ represents the feasible link flow set under the new network topology on day 1.

For the days after day 1, the proposed prediction process updates drivers’ predicted link flow pattern by:

$$\hat{x}' = (1 - \lambda_t) x^{t-1} + \lambda_t \hat{x}^{t-1} \quad \forall t > 1$$ (1)

where $\lambda_t$ is the prediction damping parameter. Eq. (1) demonstrates that drivers’ latest prediction is a weighted average of their experienced link flows and the previous prediction. This model requires $\lambda_t \rightarrow 0$ as $t \rightarrow \infty$, such that it captures the vanishing impact of prediction in drivers’ cost perception. The representation of $\lambda_t$ can vary. In this study, we let $\lambda_t = 1/t$ as an asymptotically decreasing function of time $t$. Other decreasing functions, for instance
\( \lambda_t = \lambda_0 \exp(-bt) \), can be applied as well. Given that \( \lambda_t \) is asymptotically decreasing, the predicted link flow will eventually converge to the latest actual one.

Given the predicted link flow pattern \( \hat{x}' \), the predicted link cost \( P' \) is determined by \( P' = C(\hat{x}') \). Note that the feasible link flow set \( \Omega', t > 0 \) differs from \( \Omega^0 \), in that the network topology has been changed due to the disruption.

4.2 The Correction Process

In the traditional day-to-day traffic assignment models, drivers’ perceived travel cost on day \( t \), \( F' \), is defined as a deterministic function of actual travel costs from previous days. For example, Cascetta (1989) formulated \( F' \) as a weighted average of experienced costs on days \( t-1 \), \( t-2 \), …, \( t-m \), for some finite number \( m \), as:

\[
F' = \sum_{i=1}^{m} w_i C^{t-i},
\]

where \( w_i \) is the weight of a driver’s memory of travel cost on day \( t-i \). Other studies, e.g., Davis and Nihan, 1993, assume that drivers’ perceived cost pattern is defined as a weighted average between perception and experience on previous day:

\[
F' = (1 - \alpha) F'^{t-1} + 0 \alpha C^{t-1}, \tag{2}
\]

with \( 0 < \alpha \leq 1 \) representing the cost updating weight. Eq. (2) implies an exponential weighted average of experienced costs over an infinite number of days in the past.

In our proposed model, drivers’ link cost perception is updated by:

\[
F' = (1 - \alpha) F'^{t-1} + 0 \alpha P', \tag{3}
\]

where the pre-trip link-cost prediction \( P' \) is determined by the prediction process described in the previous section. The value of \( 0 < \alpha \leq 1 \) implies the significance of prediction in drivers’ perception. Note that the prediction process appears only after a disruption occurs. If no network changes occur, then the updating process Eq. (3) gradually collapses to traditional correction process Eq. (2), due to \( P' \rightarrow C' \) when \( t \rightarrow \infty \).

In summary, the proposed perception updating process follows the “prediction-correction” framework with two steps:

\[
\hat{x}' = (1 - \lambda_t) x'^{t-1} + \lambda \hat{x}'^{t-1}, \quad P' = C(\hat{x}'); \quad \text{(Prediction Process)}
\]

\[
F' = (1 - \alpha) F'^{t-1} + 0 \alpha P'. \quad \text{(Correction Process)}
\]
Compared with traditional models, drivers’ perception combines retrospective experience $F^{t-1}$ and forward anticipation of traffic condition $P^t$ resulting from network disruption. The impact of prediction will be vanishing over time because of the damping parameter $\lambda_t$.

### 4.3 Link Flow Dynamics

With the perceived link cost at day $t$, $F^t$, the proposed day-to-day traffic assignment model employs a link-based day-to-day network loading to avoid the path-flow non-uniqueness and the path-overlapping problem (He et al., 2010). Mathematically, the link-based day-to-day dynamic is represented as:

$$x^t = x^{t-1} + \mu(y^t - x^{t-1}),$$

where $0 \leq \mu \leq 1$ is a positive constant parameter that determines the link flow changing rate. Eq. (4) means that, on day $t$, the link flow pattern tends to move from the current flow pattern $x^{t-1}$ towards a “target” flow pattern $y^t$, with a rate of $\mu$. And the target flow pattern $y^t$ is determined by solving a minimization problem:

$$\min_{y \in \Omega^t} \beta(F^t)^T y + (1 - \beta)\|x^{t-1} - y\|^2$$

where $\Omega^t = \{x : x = Af, d = \Phi f, f \geq 0\}$ is the feasible link flow set on day $t$. Objective function (5) is a weighted average of two minimization problems, $\min_{y \in \Omega^t} (F^t)^T y$ and $\min_{y \in \Omega^t} \|x^{t-1} - y\|^2$. The former is to minimize the total system cost under a given perceived cost $F^t$, which captures drivers’ cost-minimization behavior, and the latter is to minimize the distance between the current flow pattern $x^{t-1}$ and the target flow pattern $y$, which reflects drivers’ inertia or reluctance to change. Since $0 < \beta < 1$ is the weight of the cost-minimization term in objective function (5), we refer to it as the cost sensitivity parameter.

Mathematically, the minimization problem (5) is strictly convex. The strict convexity guarantees that solution $y$ is unique for any given $x^{t-1}$. Therefore, the dynamic (4)-(5) is well-defined.

Here we give a summary of the model parameters to facilitate the presentation of the model applications in subsequent sections. In its most general form, our “prediction-correction” structure involves four parameters: the prediction damping parameter $\lambda_t$ in Eq. (1), the cost
updating weight $\alpha$ in Eq. (3), the link flow changing rate $\mu$ in Eq. (4), and the cost sensitivity parameter $\beta$ in Eq. (5). Among the four model parameters, $\mu$ will be set as 1 throughout this paper to guarantee that a removed link will not carry any flow right after its removal; and $\lambda_i$ will have some predetermined functional form as discussed earlier. Therefore, in the subsequent model calibration and validation, we shall focus on two parameters only, namely $\alpha$ and $\beta$.

4.4 Stability Analysis

The stability analysis is of great importance for understanding the trajectories of day-to-day flow evolution. In the literature, Smith (1984) first analyzed the stability of a continuous day-to-day traffic evolution model using the Lyapunov theorem. Horowitz (1984) considered the stability of a discrete time day-to-day assignment model under different perception update processes. Watling and Hazelton (2003) in particular showed that a discrete time day-to-day assignment model built upon the exponential-smoothing update process (Eq. (2)) would not guarantee that a user equilibrium (UE) solution is in any sense “attractive” or “stable”. This statement induces a concern that whether a user equilibrium flow pattern is an attractive point of our proposed model with a perception update process (Eq. (3)). Although He et al. (2010) have proved that a UE flow pattern must be a fixed point of the day-to-day flow dynamic (4)-(5) without prediction process, no stability analysis is performed in that study. Since this paper puts the focus on the “prediction” component, we simply let the parameter $\alpha = 1$ throughout the stability analysis to avoid the mathematical complexities from the exponential-smoothing correction process.

In what follows, we will prove that, under mild assumptions, the proposed prediction-correction process has the user equilibrium flow pattern as a globally attractive point. We first need some definitions that are useful for analyzing the stability properties of the proposed day-to-day assignment model.

Definition 1. (Lipschitz Continuity) A mapping $f$ is said to be Lipschitz continuous with constant $L > 0$ on the set $\Omega$ if, for each pair of points $x, y \in \Omega$, we have

$$\|f(x) - f(y)\| \leq L\|x - y\|.$$  \hspace{1cm} (6)

Definition 2. (Strong Monotonicity) A mapping $f$ is said to be strongly monotone on the set $\Omega$ if, for each pair of points $x, y \in \Omega$, there exists a constant $\kappa > 0$

$$\langle x - y, f(x) - f(y) \rangle \geq \kappa\|x - y\|^2.$$  \hspace{1cm} (7)
Definition 3. (Attractiveness) The fixed point $x^*$ of link flow dynamic (4)-(5) is (globally) attractive if for any initial link flow pattern $x^0$, the Euclidean distance $\|x^t - x^*\| \to 0$ as $t \to \infty$.

Definition 4. (Stability of the System) The link flow dynamic (4)-(5) is stable if for every initial link flow pattern $x^0$, the inequality of Euclidean distance to the stable link flow pattern, $x^*$, i.e. $\|x^{t+1} - x^*\| \leq \|x^t - x^*\|$, is valid for all $t$.

Definition 5. (Asymptotic Stability of the System) The link flow dynamic (4)-(5) is asymptotically stable if it is stable and there exists an attractive link flow pattern, $x^*$, i.e., for every initial link flow pattern $x^0$,

$$x^t \rightarrow x^* \quad \text{as} \quad t \rightarrow \infty \quad (8)$$

The definitions of stability and asymptotic stability are similar to those defined in Zhang and Nagurney (1996). The following lemma will establish a connection between the minimization problem (5) and the projection operator defined as:

$$\text{Pr}_{\Omega} (x) = \arg \min_{y \in \Omega} \|y - x\|. \quad (9)$$

A basic property of the projection operator on convex set $\Omega$ is

$$\left( v - \text{Pr}_{\Omega} (v) \right)^T \left( \text{Pr}_{\Omega} (v) - u \right) \geq 0, \quad \forall v \in R^n, u \in \Omega. \quad (10)$$

Lemma 1. The minimization problem (5) projects a vector $x^{t-1} - \gamma F'$ to the current feasible link flow set $\Omega'$.

Proof: Since $x^{t-1}$ is a given link flow pattern, $F'$ is a known vector determined by (3). Then the target flow determination (5) is equivalent to:

$$\min_{y \in \Omega} \beta (F')^T (y + (1-\beta)x^{t-1} - y) \Rightarrow \min_{y \in \Omega} \|x^{t-1} - y\|^2 + 2\gamma (F')^T y - 2\gamma (F')^T x^{t-1} + \gamma^2 \|F'\|^2$$

$$\Rightarrow \min_{y \in \Omega} \|y - x^{t-1} + \gamma F'\|^2$$

By the definition of projection operator (9), above minimization problem has unique optimal solution $y = \text{Pr}_{\Omega} \left( x^{t-1} - \gamma F' \right)$ in which $\gamma = \beta / 2 (1-\beta)$.

Notice that the perception pattern $F'$ is an implicit function of the most recent link flow pattern $x^{t-1}$. Due to (1) and $\alpha = 1$, if no network changes occur after a network disruption that happens on day 0, then the prediction pattern $P' \rightarrow C \left( x^{t-1} \right)$. 

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Denote \( \mathbf{z}' = \hat{\mathbf{x}}' - \mathbf{x}' \). Due to Eq. (1),
\[
\hat{\mathbf{x}}'^{t+1} = \mathbf{x}' + \lambda_t \mathbf{z}'
\]
and
\[
\mathbf{F}^{t+1} = C(\hat{\mathbf{x}}'^{t+1}) = C(\mathbf{x}' + \lambda_t \mathbf{z}') \tag{11}
\]
with \( \lambda_t \to 0 \) when \( t \to \infty \). Then we define a residual function as:
\[
e(\mathbf{x}', \mathbf{z}'; \gamma) = \mathbf{x}' - \text{Pr}_{\Omega} \left[ \mathbf{x}' - \gamma C(\hat{\mathbf{x}}') \right] = \mathbf{x}' - \text{Pr}_{\Omega} \left[ \mathbf{x}' - \gamma C\left( \mathbf{x}' + \lambda_t \mathbf{z}' \right) \right]. \tag{12}
\]
If \( C(\hat{\mathbf{x}}'^{t+1}) = C\left( \mathbf{x}' \right) \), which is true when \( t = \infty \), the early work of Eaves (1971) has shown that a zero point of the residual function \( e(\mathbf{x}'; \gamma) = \mathbf{x}' - \text{Pr}_{\Omega} \left[ \mathbf{x}' - \gamma C\left( \mathbf{x}' \right) \right] \) solves the variational inequality (VI) problem:
\[
\left( \mathbf{x} - \mathbf{x}^* \right)^T C(\mathbf{x}^*) \geq 0, \quad \forall \mathbf{x} \in \Omega \tag{13}
\]
whose solution \( \mathbf{x}^* \) satisfies the UE principle as shown by Patriksson (1994). As \( C(\hat{\mathbf{x}}'^{t+1}) = C\left( \mathbf{x}' \right) \), He et al. (2010) have proved that, if \( \mathbf{x}^* \) is a fixed point of link flow dynamic (4)-(5), \( \mathbf{x}^* \) must be a UE flow pattern. The following lemma is important for proving the attractiveness property of the UE flow pattern.

**Lemma 2.** Assume that \( \mathbf{x}^* \) is a UE flow pattern and link performance function \( C(\bullet) \) is Lipschitz continuous and strongly monotone. If \( \gamma < 4\kappa / L^2 \), then any flow pattern \( \mathbf{x}' \) provided by the flow dynamic (4)-(5) satisfies
\[
\left( \mathbf{x}' - \mathbf{x}^* \right)^T e(\mathbf{x}', \mathbf{z}'; \gamma) \geq (1 - \delta) \left\| e(\mathbf{x}', \mathbf{z}'; \gamma) \right\|^2 - \left\| \lambda_t \mathbf{z}' \right\|^2, \tag{14}
\]
where \( \delta > 0 \) is a constant.

**Proof:** Let \( \nu = \mathbf{x}' - \gamma C(\hat{\mathbf{x}}') \) and \( u = \mathbf{x}^* \) in inequality (10), then we have
\[
\left( e(\mathbf{x}', \mathbf{z}'; \gamma) - \gamma C(\hat{\mathbf{x}}') \right)^T \left( \text{Pr}_{\Omega} \left[ \mathbf{x}' - \gamma C(\hat{\mathbf{x}}') \right] - \mathbf{x}^* \right) \geq 0. \tag{15}
\]
Notice that \( \text{Pr}_{\Omega} \left[ \mathbf{x}' - \gamma C(\hat{\mathbf{x}}') \right] \in \Omega \). Based on (13), for the UE flow pattern \( \mathbf{x}^* \), we have,
\[
\left( \text{Pr}_{\Omega} \left[ \mathbf{x}' - \gamma C(\hat{\mathbf{x}}') \right] - \mathbf{x}^* \right)^T \gamma C(\mathbf{x}^*) \geq 0. \tag{16}
\]
Adding (15) to (16) gives us
\[ \left\{ e(x', z'; \gamma) - \gamma \left[ C(\hat{x}^{t+1}) - C(x^*) \right] \right\}^T \left( \Pr_{\Omega} \left[ x' - \gamma C(\hat{x}^{t+1}) \right] - x^* \right) \geq 0. \] (17)

Due to the strong monotonicity and Lipschitz continuity of link performance function \( C(\bullet) \)

\[
0 \leq \left\{ e(x', z'; \gamma) - \gamma \left[ C(\hat{x}^{t+1}) - C(x^*) \right] \right\}^T \left( \Pr_{\Omega} \left[ x' - \gamma C(\hat{x}^{t+1}) \right] - x^* \right) \\
= \left\{ e(x', z'; \gamma) - \gamma \left[ C(\hat{x}^{t+1}) - C(\hat{x}^t) \right] \right\}^T \left[ (x' - \hat{x}^t) - e(x', z'; \gamma) \right] \\
= (x' - \hat{x}^t)^T e(x', z'; \gamma) - \| e(x', z'; \gamma) \|^2 - \gamma \left( x' - x^* \right)^T \left[ C(\hat{x}^{t+1}) - C(x^*) \right] \\
+ \gamma \left[ C(\hat{x}^{t+1}) - C(x^*) \right]^T e(x', z'; \gamma) \\
\leq (x' - \hat{x}^t)^T e(x', z'; \gamma) - \| e(x', z'; \gamma) \|^2 - \gamma \kappa \| \hat{x}^{t+1} - x^* \|^2 + \gamma \left[ C(\hat{x}^{t+1}) - C(x^*) \right]^T e(x', z'; \gamma) \\
+ \gamma^2 \| C(\hat{x}^{t+1}) - C(x^*) \|^2 + \| \lambda z' \|^2 \\
\leq (x' - \hat{x}^t)^T e(x', z'; \gamma) - \left( \frac{\gamma \kappa}{L} - \frac{\gamma^2}{4} \right) \| C(\hat{x}^{t+1}) - C(x^*) \|^2 \\
+ \gamma \left[ C(\hat{x}^{t+1}) - C(x^*) \right]^T e(x', z'; \gamma) + \| \lambda z' \|^2 \\
= (x' - \hat{x}^t)^T e(x', z'; \gamma) - \left( 1 - \frac{\gamma L^2}{4\kappa - \gamma L^2} \right) \| e(x', z'; \gamma) \|^2 + \| \lambda z' \|^2 \\
- \sqrt{\frac{\gamma L^2}{4\kappa - \gamma L^2} e(x', z'; \gamma)} - \left( 1 - \frac{\gamma L^2}{4\kappa - \gamma L^2} \right) \| C(\hat{x}^{t+1}) - C(x^*) \|\]
\[
\leq (x' - \hat{x}^t)^T e(x', z'; \gamma) - \left( 1 - \frac{\gamma L^2}{4\kappa - \gamma L^2} \right) \| e(x', z'; \gamma) \|^2 + \| \lambda z' \|^2
\]

Above inequality implies the inequality (14) with \( \delta = \frac{\gamma L^2}{4\kappa - \gamma L^2} > 0 \). \( \square \)

We can now analyze the attractiveness property of the UE flow pattern.
Theorem 1. Assume $x^{*}$ is a UE flow pattern and link performance function $C(\cdot)$ is Lipschitz continuous and strongly monotone. Also assume that the $\sum_{t=0}^{\infty} \lambda_t^2 < \infty$ and $\gamma < \frac{4\kappa}{3L^2}$. Following the “prediction-correction” structure, the UE flow pattern $x^{*}$ is a globally attractive point of traffic dynamic (4)-(5).

**Proof:** Because of Lemma 1 and Lemma 2, we have

$$
\|x^{t+1} - x^{*}\|^2 = \|\text{Pr}_\Omega \left[ x^{t} - \gamma C(x^{t} + \lambda_t z^{t}) \right] - x^{*}\|^2 = \|x^{t} - x^{*}\|^2 - 2 \langle x^{t} - x^{*}, e(x^{t}, z^{t}; \gamma) \rangle + \|\lambda_t z^{t}\|^2
$$

(18)

Since $\gamma < \frac{4\kappa}{3L^2}$ we have $1 - 2\delta > 0$. Notice that $\Omega$ is a bounded close set, thus $\|z^{*}\| = \|\lambda^{t} - x^{t}\|$ is also bounded for all $t$. Denote $a^0 = \sup_t \|\lambda^{0} - x^{0}\|^2$, then $\|\lambda^{t} - x^{0}\|^2 \leq a_0$. Sum up Eq. (18) from $t = 0$ to infinity, we have

$$
\|x^\infty - x^{*}\|^2 \leq \|x^0 - x^{*}\|^2 - \sum_{t=0}^{\infty} \left(1 - 2\delta\right) \|e(x^{t}, z^{t}; \gamma)\|^2 + \sum_{t=0}^{\infty} \lambda_t^2 a_0
$$

And since $\sum_{t=0}^{\infty} \lambda_t^2 < \infty$ we can denote $\sum_{t=0}^{\infty} \lambda_t^2 = b_0$. Thus

$$
\left(1 - 2\delta\right) \sum_{t=0}^{\infty} \|e(x^{t}, z^{t}; \gamma)\|^2 \leq \|x^0 - x^{*}\|^2 + a_0 b_0
$$

It means that

$$
\lim_{t \to \infty} e(x^{t}, z^{t}; \gamma) = \lim_{t \to \infty} e(x^{t}; \gamma) = \lim_{t \to \infty} \left[x^{t} - \text{Pr}_\Omega \left[ x^{t} - \gamma C(x^{t}) \right]\right] = 0
$$

Above limitation implies the sequence $\{x^{t}\}$ is bounded and it has a cluster point. Since the zero point of the residual function $e(x^{t}, \gamma)$ satisfies the UE principle as shown by Patriksson (1994), the UE flow pattern $x^{*}$ is a cluster point of $\{x^{t}\}$. Then the sequence $\{x^{t}\}$ has a subsequence $\{x^{j}\}$ converging to $x^{*}$. Because $e(x^{t}, \gamma)$ is continuous

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\[ e(x^*, y) = \lim_{j \to \infty} e(x^j, y) = 0 \]

In the following we prove that the sequence \( \{x^j\} \) has exactly one cluster point. Assume that \( \tilde{x} \) is another cluster point, and \( \|\tilde{x} - x^*\| = \eta > 0 \). Because \( x^* \) is a cluster point of \( \{x^j\} \), there exists a \( k_0 > 0 \) such that \( \|x^{k_0} - x^*\| < \frac{\eta}{\sqrt{8}} \). At the same time, due to \( \sum_{i=0}^{\infty} \lambda_i z^2 \leq a_i b_i \), there exists a \( k_1 > 0 \) such that \( \sum_{i=k_0}^{\infty} \lambda_i z^2 \leq \frac{\eta^2}{8} \). Without loss of generality, assume \( k_0 < k_1 \). Summing up (18) from \( t = \bar{k}_0 \) to \( t = k \) provides us

\[
\|x^k - x^*\|^2 \leq \|x_{k_0} - x^*\|^2 - \sum_{i=k_0}^{k} (1 - 2\delta) e(x', z'; y) + \sum_{i=k_0}^{k} \lambda_i z^2 \\
< \|x_{k_0} - x^*\|^2 + \frac{\eta^2}{8} \leq \frac{\eta^2}{4} \quad \forall k \geq k_0
\]

Above inequality implies \( \|x^k - x^*\| < \frac{\eta}{2} \), for all \( k > k_0 \). By applying triangle inequality, it follows that

\[
\|x^k - \tilde{x}\| \geq \|\tilde{x} - x^*\| - \|x^k - x^*\| \geq \frac{\eta}{2}, \quad \forall k \geq k_0
\]

This is a contradiction to the assumption that that \( \tilde{x} \) is a cluster point of the sequence \( \{x^j\} \). Thus \( x^* \) globally converges to the UE flow pattern \( x^* \) as \( t \to \infty \). In other words, the UE flow pattern \( x^* \) is a globally attractive point of traffic dynamic (4)-(5).

**Remark 1.** Theorem 1 provides sufficient conditions to the global attractiveness property of the UE flow pattern for the traffic dynamic (4)-(5). A specific sufficient condition is that of \( \sum_{i=0}^{\infty} \lambda_i^2 < \infty \). This sufficient condition can easily be satisfied, e.g., \( \lambda_i = 1/t \). However, the traffic dynamic (4)-(5) may not converge to the UE flow pattern, if \( \lambda_i \) is a constant. Instead, it may stabilize at a point that differs from the UE flow pattern, when the impact of prediction does not disappear quickly enough.
Remark 2. Another sufficient condition for the global attractiveness requires that \( \gamma < \frac{4\kappa}{3L^2} \). Notice \( \gamma = \beta/2\left(1 - \beta\right) \). This condition implies \( \beta < \frac{8\kappa}{(3L^2 + 8\kappa)} \), which offers a threshold value of \( \beta \) to guarantee the attractiveness. As we will see, a large \( \beta \) may result in divergent trajectories of the traffic dynamic (4)-(5). However, due to the difficulty of estimating scalars \( \kappa \) and \( L \), this sufficient condition is generally difficult to be verified.

Remark 3. From the proof, we can see that the link flow traffic dynamic (4)-(5) is not stable, due to the existence of the predicted traffic pattern \( \lambda_i z_i \) in travelers’ perception. Even though the initial traffic flow pattern \( x^0 \) is \( x^* \), the traffic pattern \( x' \) may be diverted away from \( x^* \). At the time immediately after an unexpected network disruption, \( \lambda_i z_i \) has a significant impact to the system, such that the inequality \( \|x'^{t+1} - x^*\| \leq \|x' - x^*\| \) may not be valid. However, the stability property can be developed easily if the prediction process is removed, as shown by the following corollary.

Corollary 1. Assume \( x^* \) is a UE flow pattern and link performance function \( C(\bullet) \) is Lipschitz continuous and strongly monotone. Also assume that \( \gamma < \frac{2\kappa}{L^2} \). If \( \hat{x}' = x' \) or \( \lambda_i \equiv 0 \) then the link flow traffic dynamic (4)-(5) is asymptotically stable.

Proof: As \( \hat{x}' = x' \) or \( \lambda_i \equiv 0, \lambda_i z_i \equiv 0 \). Due to (18), we have

\[
\|x'^{t+1} - x^*\|^2 \leq \|x' - x^*\|^2 - (1 - 2\delta)\|C(x', \gamma)\|^2.
\]

Since \( \gamma < \frac{2\kappa}{L^2} \), \( 1 - 2\delta > 0 \). Above inequality shows that link flow traffic dynamic (4)-(5) is stable. Theorem 1 has proved that \( x' \) globally converges to \( x^* \) as \( t \to \infty \). Thus, the link flow traffic dynamic (4)-(5) is asymptotically stable. \( \square \)

Remark 4. This corollary indicates that, by removing the prediction process, the traffic flow pattern will approach to the new UE flow pattern asymptotically. On the contrary, the existence of the prediction process offers the possibility that the traffic flow may divert away from the UE flow pattern. That allows us to capture the traffic recovery characteristics shown in the field observations.
4.5 An Illustrative Example

Before we apply the proposed model on the Twin Cities network, in this section, we will demonstrate the modeling flexibility of the “prediction-correction” structure using a simple network with disruption. Suppose that we have a small network, shown in Figure 6, with 8 nodes, 11 links, and 2 origin-destination (OD) pairs: (1, 3) and (2, 4). Both OD pairs have the same OD demand of 1000 trips. All links in this network have the same capacity of 100, and their link performance functions are of the Bureau of Public Roads (BPR) type:

\[
t_a(x_a) = t^0_a \left[1 + 0.15 \left( \frac{x_a}{c_a} \right)^4 \right],
\]

where \( t^0_a \) represents the free flow travel time on link \( a \); \( x_a \) and \( c_a \) are link flow and link capacity on link \( a \), respectively. Links #4, #5, #6, #7, #8 have a free flow travel time of 5, while other links have a free flow travel time of 20.

Assume that the link flow pattern follows user equilibrium before link #9 is removed at the end of day 1. The detour route is designated as link chain #7->#6->#8 after the removal of link #9 since this route has shortest free flow travel time. Therefore, the impacted traffic using link #9 is redistributed to the designed detour in the prediction process. The output of the prediction process generates a predicted traffic pattern \( \hat{x}^2 \). As mentioned earlier, we set link flow update parameter \( \delta = 1 \) in Eq.(4); otherwise, some traffic will remain on link #9 on day 2 even though it has been removed. We set parameter \( \xi = 1/(t-1) \) in Eq. (1), as discussed before, to capture the damping effect of prediction. For comparison purposes, we test the proposed model under different settings of \( \alpha \) and \( \beta \).

In the first trial, we investigate the impact of the prediction weight parameter \( \alpha \) on the day-to-day flow evolution after network disruption. To do so, we set different values of \( \alpha \), while maintaining a constant cost sensitivity of \( \beta = 0.3 \). Figure 7 shows that the link flow on link #6
gradually stabilizes in all cases. One important feature we can observe from the figure is that as the value of $\alpha$ increases the link flow on link #6 drops more significantly. The reason is that the prediction process in the proposed model puts more weight on drivers’ consideration about the potential congestion on the designated detour, as a consequence of the removal of link #9.

![Figure 7 Flow Evolutions on Link #6 with Different Perception Updating Weights](image)

The cost sensitivity parameter $\beta$ also plays an important role in the stability of traffic evolution. In the second trial we keep $\alpha$ as a constant ($\alpha = 0.6$) and let traffic evolve by using different values of $\beta$. The link flow evolution patterns shown in Figure 8 provide an interpretation of the system stability: A larger value of $\beta$ (e.g., $\beta = 0.6$ in this trial) would widen the variation of link flows, because higher sensitivity to travel cost would result in more drastic changes of their daily travel choice. This is also a demonstration of Remark 2. Whenever $\beta$ violates the threshold value, e.g. $\beta < \frac{8\kappa}{\left(3L^2 + 8\kappa\right)}$ shown in Theorem 1, the attractiveness property of the link flow dynamic cannot be guaranteed anymore.

![Figure 8 Flow Evolutions on Link #6 with Different Cost Sensitivity Parameters](image)
In the final trial, we demonstrate that it is essential to include the prediction process in modeling traffic evolution of a disrupted network. This example is an illustration of Corollary 1. Specifically, we remove the prediction process from the day-to-day traffic assignment process by letting $\lambda_i = 0$ in Eq. (1), such that the process coincides with traditional day-to-day assignment models. By doing so, drivers’ link cost perception on day 2 (i.e. immediately after the removal of link #9) is the same as the pattern before the link removal. As shown in Figure 9, if no prediction appears in drivers’ perception, traffic volume on link #6 on day 2 always increases, since users of link #9 would always consider the detour as a main alternative. A decreased link flow on link #6 would never happen, which is certainly possible because of drivers' anticipated congestion (see Figure 2 for the case of the I-35W Bridge collapse). Therefore, the existence of the prediction process offers us more flexibility to capture the traffic flow evolution characteristics after disruption.

5. **Model Calibration and Validation: The Collapse of I-35W Mississippi River Bridge**

To demonstrate the applicability of the proposed model, we validate it against the field data collected before and after the I-35W bridge collapse. Before collapse, more than 140,000 vehicles crossed the I-35W Bridge daily. As a major entry of city Minneapolis, this bridge carried nearly 25% of the traffic accessing the downtown area.

The network used in our work is the Twin Cities Seven-County conflated planning network. This network contains 22,476 links, and 8,618 nodes, of which 1201 are traffic analysis zones (TAZs) generating and absorbing trips. A trip table, derived from 2006 Longitudinal Employer-
Household Dynamics (LEHD) database, is adopted as the origin-destination demand data. The projected daily demands in 2006 LEHD were scaled to reflect trip-desires during morning peak hours in 2007. A demand multiplier was determined to make the traffic assignment result as close as possible to the observed data.

The study period chosen in this study is from July 30, 2007 to August 31, 2007. Only weekdays are considered. A series of network topology changes took place during this period. In detail, the I-35W Bridge collapsed on August 1, 2007, and freeway ramps around the bridge were closed; after the collapse, the Cedar Ave Bridge (an urban arterial parallel to the I-35W Bridge) remained closed for investigation purpose until August 31; TH 280 was approved as a detour route and was converted to a freeway by blocking off all side-street access on August 2; further, the ramps connecting TH 280 and I-94 were expanded to two lanes on August 13; finally, on August 20, one additional lane was added on each direction of I-94 for the three-mile stretch between TH 280 and I-35W to accommodate re-routed traffic.

We calibrated the proposed model by using the 15 weekdays’ (July 30, 2007 to August 17, 2007) loop detector data collected from 10 detector stations. The objective of the parameter calibration is to minimize the root mean square percent error (RMSPE):

$$\min_{\theta} l(\hat{\theta}) = \frac{1}{n} \sum_{t} \sum_{i} \left[ z_i^t - \hat{z}_i^t(\hat{\theta}) \right]^2,$$

where $z_i^t = (x_i^t - x_i^0)/x_i^0$ represents the observed link-flow-change percentage w.r.t day 0 (July 30, 2007), on link $i$. The reason for applying root mean square percent error is that we attempt to model the day-to-day link flow evolution pattern instead of the absolute values of link flows. The percentage change of link flow is appropriate for this purpose, since it considers the relative flow changes and mitigates the impact from the difference between the initial estimated link flow pattern and field observations. The predicted link-flow-change percentage $\hat{z}_i^t = (\hat{x}_i^t - \hat{x}_i^0)/\hat{x}_i^0$ is determined by the proposed day-to-day traffic assignment model under given parameters $\hat{\theta} = (\hat{\alpha}, \hat{\beta})$. And $n = 140$, since we applied data from 10 detector locations, each of which provides 14 days’ link-flow-change percentages.

We applied a mesh method to estimate the parameters in the model, taking the other parameter $\lambda_0 = 0$ when $t \leq 3$ (i.e. no prediction before the collapse) and $\lambda_t = 1/(t-3)$ when $t > 3$. First, we divided the two-dimensional parameter space into small bins, using a step size of 0.1 along each axis. Thus, we had ten bins along each parameter domain and a grid of 100 bins.
We randomly generated one sample inside each bin and obtained total 100 sampled pairs of parameters. Each pair of parameters \( (\hat{\alpha}, \hat{\beta}) \) is applied with the proposed day-to-day traffic assignment method to generate daily traffic flow patterns. And then, we calculated the RMSPE for each day-to-day assignment result.

We found that the sampled parameters \( \hat{\alpha} = 0.304 \) and \( \hat{\beta} = 0.902 \) provide the minimal value of the RMSPE. With the calibrated parameters, we then compared the model results with the field observations for the inbound traffic volumes crossing the three cordons.

![Figure 10](image.png)

Figure 10 Inbound Traffic Volumes Crossing the Three Cordons from the Day-to-Day Traffic Assignment Model (with Prediction Process)

Figure 10 shows the day-to-day traffic fluctuations from July 30, 2007 to August 31, 2007 on the three cordons defined in Section 2. Compared with the field observations, shown in Figure 2, the traffic recovery process after the collapse of I-35W Bridge is replicated by the proposed “prediction and correction” day-to-day traffic assignment model. The traffic recovery process generated from our model has similar characteristics with the one observed from the field.

To demonstrate that it is essential to include the prediction process in modeling a disrupted network, we also applied the day-to-day traffic assignment framework without the prediction process. By setting the prediction parameter \( \lambda_t = 0 \) in our proposed model, we repeated the same parameter estimation methodology to calibrate the parameters in the day-to-day assignment model.
Without prediction, $\hat{\alpha} = 0.1928$ and $\hat{\beta} = 0.8924$ provide the minimal value of RMSPE from the 100 random samples. Then we had another set of day-to-day traffic assignment results using the calibrated model without the prediction. We summed up the total volume entering into the defined cordons, and obtained the day-to-day traffic fluctuations on the three cordons, illustrated by Figure 11.

![Figure 11 Inbound Traffic Volumes Crossing the Three Cordons from the Day-to-Day Traffic Assignment Model (without Prediction Process)](image)

The quality of the proposed day-to-day traffic assignment model can be verified by comparing the volume percentage changes on cordons with field observations. To do so, we denote $u^t_i$ as the hourly volume entering cordon $i$ on day $t$, $u^0_i$ as the hourly volume entering cordon $i$ on the first day (July 30, 2007), and $p^t_i = u^t_i / u^0_i$ as the cordon $i$’s volume percentage with respect to the first day’s cordon volume. The comparisons of volume percentage changes on cordons are summarized into the following figures.
Figure 12 Comparisons of Volume Percentage Changes on Cordons

(a) Volume Percentage Changes on Cordon 1

(b) Volume Percentage Changes on Cordon 2

(c) Volume Percentage Changes on Cordon 3

Figure 12 Comparisons of Volume Percentage Changes on Cordons
Based on the figures shown above, we can observe that the prediction component plays a key role in replicating the traffic restoration process after an unexpected network disruption. The cordon volume percentage changes given by the model with prediction process fit much better with the observations. The traffic recovery process on Cordon 2 and Cordon 3 is clearly captured by the proposed day-to-day traffic assignment model.

6. CONCLUSIONS

In this paper, we propose a “prediction-correction” framework for modeling discrete-time deterministic day-to-day traffic evolution. The proposed modeling framework provides the flexibility to accommodate drivers’ forward-looking behavior in response to a network disruption. It has been rigorously proved that the proposed day-to-day link flow evolution model has the user equilibrium flow pattern as a globally attractive point. The proposed model has been calibrated and validated using the field data collected before and after the I-35W Bridge collapse in Minneapolis, Minnesota.

For future research, this study can be extended in several directions. First of all, the stability properties of the system deserve further analysis. The attractiveness and stability of a day-to-day traffic evolution system impact its practical applications. The attractiveness property provided in this paper has been restricted by letting \( \alpha = 1 \). A stability analysis is desired to accommodate the more general exponential-smoothing perception update process. Secondly, developing stochastic models could be a direct extension of this research; and demand elasticity after disruption can also be considered in the “prediction-correction” framework. In addition, the predicted traffic flow pattern in the prediction process can have various representations, under different perception assumptions, all of which need empirical verifications. For example, the anticipation of traffic congestion can be assumed to be a function of the severity of the disruption and the distance from the disruption location. Finally, it is possible to formulate a day-to-day network design model to help traffic engineers develop effective traffic restoration projects, incorporating the proposed “prediction-correction” traffic assignment approach.

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