Governing equation for the Mean Flow

after applying Boussinesq approx., we still have an equation for instantaneous quantities (including $\theta'$)

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\delta_{i3} \left[ g - \left( \frac{\theta'}{\theta_v} \right) g \right] + f_c \varepsilon_{ij3} u_j - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

This was the small price to pay for abandoning the equation of state for instantaneous quantity.

Now we can decompose this simplified form of the momentum equation using mean and fluctuating components $u = U + u'$ (same for $\theta$, but not $p$ or $\rho$) and focus on the mean quantities. Note that the only non-linear term is $u_j \frac{\partial u_i}{\partial x_j}$

$$(U_j + u_j') \frac{\partial (U_i + u_i')}{\partial x_j} = U_j \frac{\partial U_i}{\partial x_j} + u_j' \frac{\partial U_i}{\partial x_j} + U_j \frac{\partial u_i'}{\partial x_j} + u_j' \frac{\partial u_i'}{\partial x_j}$$

So far we did not lose any information. Let us average now!

$$U_j \frac{\partial U_i}{\partial x_j} + u_j' \frac{\partial u_i'}{\partial x_j} = U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial u_i' u_j'}{\partial x_j}$$

we must retain the derivative of the Reynolds stresses... $1/\rho \frac{\partial (\tau_{ij}^{Reynolds})}{\partial x_j}$
Important: turbulence affects the mean flow in the equation. We have a closure problem. The mean flow cannot be predicted without accounting for turbulence: we need an equation for $u'_i u'_j$ ... or for $u'_i$
Governing equation for the Mean Temperature or conservation of Heat

\[
\frac{\partial \bar{\theta}}{\partial t} - u_j \frac{\partial \bar{\theta}}{\partial x_j} = \nu_\theta \frac{\partial^2 \bar{\theta}}{\partial x_j^2} - \frac{\partial}{\partial x_j} \left( u_j' \theta' \right) + Q
\]

I  Mean storage of heat
II  Advection of heat by mean wind
III  Mean molecular conduction of heat
IV  Divergence of TURBULENT HEAT FLUX
V   Sources/sinks of heat

Important: turbulent heat flux affects the mean temperature. Again a closure problem.
Shallow motion approximation, neglecting subsidence \((W\sim0)\)

\[ W=0 \text{ (negligible mean vertical velocity)} \]

hydrostatic pressure \( \frac{\partial \bar{p}}{\partial z} = -\bar{\rho}g \)

this is what remains of the vertical mean velocity component neglecting vertical motion and Reynolds stress terms (negligible drag!)

the remaining two velocity components of interest are \(U\text{(east)}\) and \(V\text{(north)}\) as a function of \(x,y\). The driving forces are Coriolis and pressure gradients.

**Geostrophic wind and geostrophic balance**

Note that following this balance, the Press. grad terms is replaced by a geostrophic velocity term.
The geostrophic wind is a reasonable model but in some cases it doesn’t describe exactly what we observe in nature (→Fig.2). Friction forces near the earth surface exert a drag on the flowing air. The air slows down and therefore the Coriolis deflection decreases. Now, the pressure gradient force becomes more dominant and the wind gets a component in the direction of low pressure.

Fig. 2: Non-geostrophic air flow to low pressure
Summary of the mean flow equation, following Boussinesq AND shallow convection AND subsidence approx. these 3 assumptions are related but strictly independent

1) ideal gas
   \[ \frac{\partial \rho}{\partial t} = -\rho \frac{\partial T}{\partial x} \]

2) continuity
   \[ \frac{\partial U_j}{\partial x_j} = 0 \]

3) replace the pressure gradient term with the (virtual)geostrophic wind components (assuming balance)

4) viscous terms are neglected as the Reynolds number is high

5) what happened to the equation for \( W \)?
   \( W = 0 \) \( \to \) hydrostatic pressure
   this is the core of the shallow convection approx.

6) turbulence affect the mean flow through the divergence of turbulent fluxes

note that the solution of this eqs is e.g. \( U(x,y,z) \) or \( \theta(x,y,z) \)
Daytime boundary layer
(turbulent fluxes > 0 at the surface, <0 below the mixed layer interface)

- Surface heating by the sun
- Entrainment of “warm air” from the free atmosphere
- The net vertical heat flux is towards the surface
- Evaporation of moisture at the surface

\[ \theta' > 0, \ w' < 0 \text{ so } \theta' w' < 0 \]

Example BB

Fig. 3.3 (a) Kinematic heat flux \( (w'\theta') \) and moisture flux \( (w'\rho'_v) \) at various heights measured by aircraft during flight 13, June 14, of the 1983 Boundary Layer Experiment (BLX83) in Oklahoma, where \( \rho_v \) is the water vapor density \( (g_{\text{water}} \text{ m}^{-3} \text{ air}) \). (b) Same data as (a), but normalized with respect to the top of the mixed layer \( (z_i) \), and with respect to surface values of the fluxes. The buoyancy flux \( (w'\theta' v') \) is also plotted.
Daytime boundary layer

note the increased size of the mixing layer, and the shifting of the inversion layer

**Fig. 3.7** Daytime temperature evolution as a function of local time. Modeled (left) and observed (center) profiles of mean virtual potential temperature during Day 33 of the Wangara field experiment. Computed (right) profiles of kinematic heat flux during Day 33. The computed profiles (André, et al., 1978) show the idealized shape and evolution.
Daytime boundary layer (2)

note the increased size of the humidity in the mixing layer, the profile becomes steeper

Fig. 3.8 Modeled (left) and observed (right) profiles of mean humidity during Day 33 for Wangara, valid at the local time indicated (André, et al., 1978).
Neutral boundary layer

in neutral conditions \(<u'w'\>) defines the shear velocity.
Nocturnal boundary layer

Radiative cooling of the surface influence the temperature profiles more than the turbulent heat flux.

Radiation Inversion

Temperature reduces with height above the inversion layer

Subsidence: cooler air tend to squeeze down the trapped layer of warm air

Warmer layer of air above remains unaffected by the cold surface

Lower layer cools by contact with the cold surface