Estimate of TKE and temperature variance dissipation in the stable boundary layer

1) Using directly the velocity timeseries:
   Taylor 1935
   \[ \varepsilon = \frac{15 \nu}{U^2} \left( \frac{\partial u}{\partial t} \right)^2 \]
   Dependent on the sampling frequency
   No actual quality checks available

2) Using the premultiplied spectrum
   Of the streamwise velocity fluctuation
   Batchelor (1953) see Saddoughi and Veeravalli 1994
   \[ \varepsilon = 15 \nu \int_0^\infty k_1^2 E_{11}(k_1) \, dk_1 \]

\( \nu \) is the kinematic viscosity
Note that the procedure is iterative,
1) Calculate the integral of the dimensional premultiplied spectrum
2) Estimate the dissipation
3) Normalize the spectrum and verify the dimensionless energy level

Saddoughi and Veeravalli 1994

Figure 10. Dissipation spectra measured at mid-layer for the low-speed case ($y = 515$ mm, $R_A \approx 600$). (a) $u_1$-spectrum; (b) $u_2$-spectrum; (c) $u_3$-spectrum.
This is the actual extrapolation of the inertial range to the limit of finite dissipation.
Note that there is no indication of a dissipative range in the K41 assumptions and derivation.

Figure 9. Kolmogorov's universal scaling for one-dimensional longitudinal power spectra. The present mid-layer spectra for both free-stream velocities are compared with data from other experiments. This compilation is from Chapman (1979), with later additions. The solid line is from Pao (1965), $R_e$, $\Box$, 23 boundary layer (Tielman 1967); $\bigcirc$, 23 wake behind cylinder (Uberoi & Freymuth 1969); $\bigtriangledown$, 37 grid turbulence (Comte-Bellot & Corrsin 1971); $\bigtriangledown$, 53 channel centreline (Kim & Antonia (DNS) 1991); $\blacklozenge$, 72 grid turbulence (Comte-Bellot & Corrsin 1971); $\bigcirc$, 130 homogeneous shear flow (Champagne et al. 1970); $\blacksquare$, 170 pipe flow (Laufer 1954); $\phi$, 282 boundary layer (Tielman 1967); $\blacklozenge$, 308 wake behind cylinder (Uberoi & Freymuth 1969); $\triangle$, 401 boundary layer (Sanborn & Marshall 1965); $\blacktriangle$, 540 grid turbulence (Kistler & Vrebalovich 1966); $\times$, 780 round jet (Gibson 1963); $\blacklozenge$, 850 boundary layer (Costant & Favre 1974); $+$, 2000 tidal channel (Grant et al. 1992); $\bigcirc$, 3180 return channel (CAHI Moscow 1992); $\bullet$, 1500 boundary layer (present data, mid-layer: $U_\tau = 50$ m s$^{-1}$); $\blacksquare$, 600 boundary layer (present data, mid-layer: $U_\tau = 10$ m s$^{-1}$).
Verify the shape and the normalized energetic level of this 1D spectrum:

\[ C = 1.5, \quad C_1 = (18/55) C \]

\[ C_1 = (4/3) C_1 \]

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C is the Kolmogorov constant. In the original K41

Note that this check is particularly tough to pass: the y axis is linear!
3) Using the second order structure function, with empirical coefficients

\[ D_{11}(r) \equiv [u_1(x_1 + r) - u_1(x_1)]^2 = C_2 C^{3/5} r^{3/5} \]  
\[ D_{33}(r) = D_{22}(r) \equiv [u_2(x_1 + r) - u_2(x_1)]^2 = C_2' C^{3/5} r^{3/5} \]

respectively in isotropic flow, where \( C_2 \approx 4C_1 \) (Monin & Yaglom 1975) and \( C_2'/C_2 = 4/3 \). This structure-function behaviour is also known as ‘Kolmogorov’s \( \frac{3}{5} \) law’.

Note that \( C_1 = 18/55 \) and \( C = 1.5 \)

The estimate can be obtained also using the structure functions in the spanwise or vertical velocity component \( D_{22} \) and \( D_{33} \) (still as a function of \( r \) for simplicity in the streamwise direction > apply Taylor).
3) Using the third order structure function, with exact coefficient (but more difficult to converge)

\[ D_{111}(r) \equiv \left[ u_1(x_1 + r) - u_1(x_1) \right]^3 = -\frac{4}{5} \epsilon r. \]

**Figure 16.** Compensated third-order structure functions for longitudinal velocity fluctuations measured at mid-layer. (a) Low-speed case \((y = 515 \text{ mm}, y^+ \approx 16200, R_e \approx 600)\); (b) high-speed case \((y = 400 \text{ mm}, y^+ \approx 62000, R_e \approx 1450)\).
Estimating the dissipative term in the temperature variance equation.

1) Using the premultiplied spectrum of the temperature fluctuation

If \( \theta \) is the deviation of a scalar property from its ‘ensemble’ mean, the spectral functions for its fluctuations are defined so that

\[
\overline{\theta^2} = \int_0^\infty E_\theta(k) \, dk, \quad \overline{\theta^2} = \int_0^\infty \phi_\theta(k_1) \, dk_1, \tag{9}, (10)
\]

where \( E_\theta(k) \) is the three-dimensional spectrum, and \( \phi_\theta(k_1) \) is the one-dimensional spectrum. These spectra are related in the isotropic case (Hinze 1959, p. 226) by

\[
E_\theta(k_1) = -k_1 \frac{\partial \phi_\theta(k_1)}{\partial k_1}. \tag{11}
\]

The mean rate of scalar dissipation \( \epsilon_\theta \) is determined by

\[
\epsilon_\theta = 2\kappa \int_0^\infty k^2 E_\theta(k) \, dk = 6\kappa \int_0^\infty k_1^2 \phi_\theta(k_1) \, dk_1, \tag{12}
\]

Note that \( k \) here is the thermal diffusivity of air.

The Prandtl number is defined as \( \text{Pr} = \frac{\nu \text{ (kinematic viscosity)}}{\kappa \text{ (thermal diffusivity)}} \).

For standard air \( \text{Pr} \sim 0.7-0.8 \) and \( \nu = 1.5 \times 10^{-5} \), usually \( \kappa = 1.9 \times 10^{-5} \text{ m}^2/\text{s} \).
Note that here $k_s$ is the Kolmogorov wave number

$$K_s = \frac{1}{\eta}$$

Note that $\log_{10}(1) = 0$
It is consistent with the Previous graph
Same procedure as before:
1) Estimate thermal dissipation on the dimensional spectrum
2) Use your estimate to normalized the spectrum
3) Check the dimensionless energy level

Note that here $k_s$ is the Kolmogorov wave number $K_s = 1/\eta$
Using mixed third order structure functions

\[ D_{u\theta\theta}(r) = -\frac{4}{9} \langle \varepsilon_\theta \rangle r \quad \text{where} \quad D_{u\theta\theta}(r) = \left\langle \left( u(x + r) - u(x) \right) \cdot \left( \theta(x + r) - \theta(x) \right)^2 \right\rangle \]